Shape analysis of MR brain images based on the fractal dimension

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Abstract
This paper presents some results concerning the application of the fractal approach to the analysis of shape of white brain matter in the magnetic-resonance (MR) images. The fractal dimension of white brain matter was calculated using the box-counting algorithm.

1. Introduction

The fractal, introduced in 1975 by Mandelbrot [1], provides a framework for the analysis of natural phenomena in various scientific domains. The fractal is an irregular geometric object with an infinite nesting of structure of different sizes. Fractals can be used to make models of any natural object, such as islands, rivers, mountains, trees, clouds, or snow flakes. The most important properties of fractals are self-similarity, chaos and non-integer fractal dimension. Fractals are self-similar, which means that copies of themselves can be found in different scales of size. The fractal dimension itself is often considered as a parameter describing the morphological complexity of objects.

The fractal dimension analysis has been applied to study a wide range of objects in biology and medicine. The fractal dimension analysis method has been successfully used to diagnose blood cells [2], human cerebellum [3], to detect small peripheral lung tumors [4], micro-calcification in mammograms [5], and tumors in brain [6]. The investigators in these studies concluded, based on their observations, that the changes in the fractal dimension value reflect alterations of structural properties.

In this study, we propose a modified box-counting fractal dimension algorithm to analyze the shape of white brain matter in MR images.

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2. Fractal Dimension

Euclidean geometry describes lines, planes and cubes. Euclidean geometry works with objects, which exist in integer dimensions as single dimensional points, one dimensional lines and curves, two dimension plane figures like circles and squares, and three dimensional solid objects such as spheres and cubes. Mandelbrot [1] used the term "topological dimension" to describe these shapes. However, many complex objects in nature are described better with the fractal dimension as a non-integer value that lies strictly in Euclidean space, being somewhere between two whole numbers. While a straight line has a dimension of exactly one, a fractal curve will have a dimension between a straight line and a plane (between one and two), depending on how much space it takes up as it curves and twists. A fractal surface will have dimension between a plane and a three-dimension space (between two and three). A fractal object possesses a fractal dimension strictly greater than its topographical dimension. Several definitions of fractal dimension have been proposed, such as the Hausdorff-Besicovitch dimension, the Bouligand-Minkowski dimension, the Tricot dimension, and others [7]. In practice, Mandelbrot [1] has popularized the Hausdorff-Besicovitch dimension or mass dimension (as the measure is very often a mass), which happens to be one of the simpler and more understandable dimensions. The equation for the Hausdorff-Besicovitch dimension, D_H is defined as:

$$D_H = \lim_{r \to 0^+} \frac{\ln N}{\ln \left(\frac{1}{r} \right)}, \qquad (1)$$

where N is the number of self-similar pieces, with the magnification factor, $1/r$, into which a figure may be broken.

3. Box counting method for estimating the fractal dimension

The box counting method is one of the wide variety algorithms for estimating the fractal dimension of a fractal object. It determines the fractal dimension of black and white images of fractals. It works by covering image with boxes and then evaluating how many boxes are needed to cover the fractal completely. Repeating this measurement with different sizes of boxes r will result into the logarithmical function of box size $(\ln 1/r,$ x-axis) and the number of boxes needed to the cover fractal $(\ln N,$ y-axis). The fractal dimension of the fractal object is estimated by the slope of points ($\ln N$ versus $\ln 1/r$), which normally lies on a straight line.

4. Fractal dimension algorithm

The fractal image-analysis program is developed in C++ Microsoft Visual Studio NET 2003. The first step for fractal image analysis of brain MR images

consists in segmentation of white brain matter. The white brain matter was segmented by thresholding (threshold of about 70 units) to isolate and refine the white brain matter region image being labelled, the connectivity style, using the seeded region growing method. The seeded region growing method is based on eight-neighborhood connectivity. That sorts the individual components according to size and keeps only the largest component, labelling it as white (index 255). The background of the object is labelled as black (index 0). Fig. 1 shows the brain MR image and the image after segmentation of white brain matter.

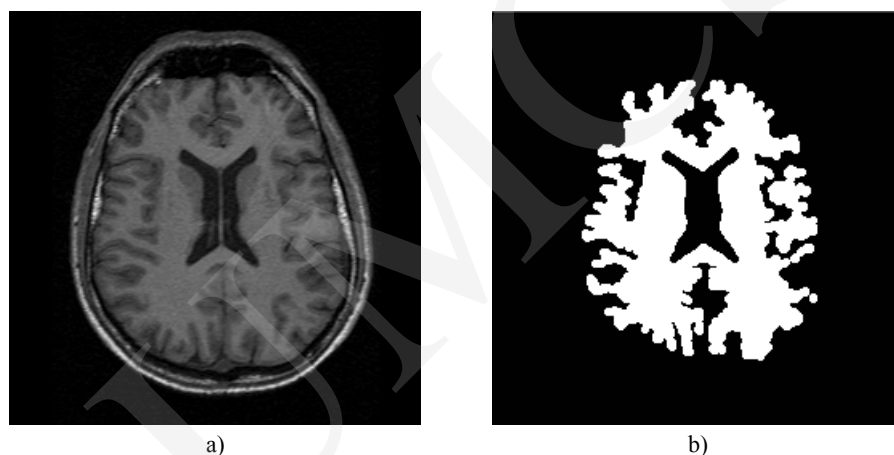a)   b)

Fig. 1. (a) Brain MR image and (b) segmented image of white brain matter

For the fractal image analysis we propose a modification of the traditional box counting method. By this modification we obtain not one, but three fractal dimensions. We count:
- white boxes that cover the object and boxes which cover the border of white object (traditional method) (FD1),
- separately, boxes that cover the border of the white object (those boxes which contain at least part of the tested white object) (FD2),
- boxes that contain just black background (FD3).

The box sizes in the program are entire from the first to the last and for the entire step. The digitized image is the first scanned from the left to the right and from the top to the bottom to find the count number (N) in the response box size (r). Fig. 2 presents the images below indicating the counting process.

The results of Estimation Fractal Dimension of white brain matter in MR images are presented in Table 1. Where r – the box size (in pixels), N1(r) – the number of boxes that cover the object and the boxes which cover border of white object, N2 (r) – the number of boxes containing the border of object, N3(r) – the number of boxes containing the background.
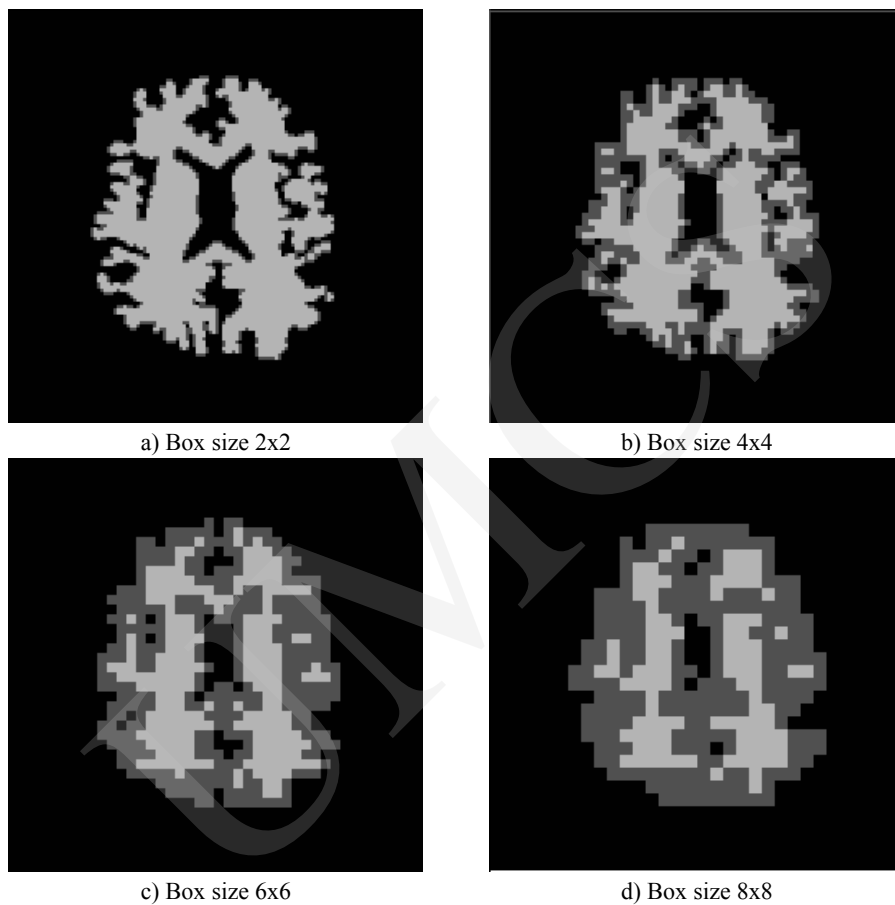
a) Box size 2x2

b) Box size 4x4

c) Box size 6x6

d) Box size 8x8

Fig. 2. Images below show the counting process

Table 1. The results of Estimation Fractal Dimension white brain matter in MR images

r	N1(r)	N2(r)	N3(r)	ln(1/r)	ln(N1(r))	ln(N2(r))	ln(N3(r))
2	7213	3932	12452	-0.69314	8.88364	8.27690	9.42963
4	1789	1129	2967	-1.38629	7.48941	7.02908	7.99530
8	430	326	698	-2.07944	6.06378	5.78689	6.54821
16	106	96	160	-2.77258	4.66344	4.56434	5.07517
32	30	30	34	-3.46573	3.40119	3.40119	3.52636
64	12	12	4	-4.15888	2.48490	2.48490	1.38629

Using a regression method we find the lines that fit the points of ln N versus ln $(1/r)$. Fig. 3 shows the lines that have been fitted to the points of ln N versus ln $(1/r)$. The fractal dimension is the slope of these lines, in this case FD1 = 1.88204, FD2= 1.69274, FD3=2.27107.

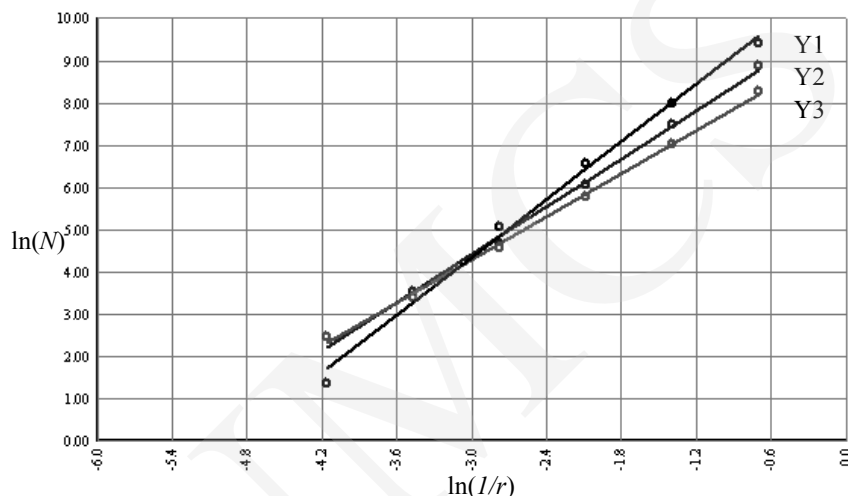Fig. 3. Lines that are fitted to the points of ln N versus ln $(1/r)$
Y1=1.88204x + 10.06359, Y2=1.69274x + 9.36384, Y3= 2.27107x +11.16982

5. Conclusions

Fractal geometry could be useful for the characterization of brain MR images due to their highly complex structure. Fractal dimensioning allows us to measure the degree of complexity by evaluating how fast our measurements increase or decrease as our scale becomes larger or smaller.

In this study, we discuss the modified box-counting fractal dimension algorithm for shape analyses of white brain matter in MR images. The box counting dimension is much more extensively used than the other dimension since the box-counting dimension can measure objects that lack strict self-similarity. It is known that most real-life objects are not self-similar.

For the analysis of white brain matter shapes we obtained fractal dimensions of 20 normal brain MR images. The fractal dimensions were FD1= 1.887 ± 0.008, FD2=1.696 ± 0.008, FD3=2.272 ± 0.008.

In the future, we want to implement other algorithms for calculating the fractal dimension of white brain matter in MR images using the "hand and dividers" method, based on the idea of measuring the fractal dimension via perimeter estimation with different yardstick sizes.

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