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Impact of Macroeconomic Variables on Tax Revenues – Empirical Study Using the VEC Model

Wpływ zmiennych makroekonomicznych na dochody podatkowe – badanie empiryczne przy użyciu modelu VEC

Introduction

An economy consists of close links between households, entrepreneurs and the state, such as the tax revenues that may affect this system through economic policy. Tax policy might be viewed as an indispensable element of the economic policy of a given country for maintaining and strengthening economic growth and global competition. It also provides countries with a stable predictable fiscal environment, making it possible for them to gather funds to finance their social and infrastructural needs [Romer, Romer, 2010, p. 764]. Taking into consideration the importance of tax revenues, it is necessary to investigate the relationships between them and other microeconomic factors.

Tax is an obligatory payment levied by public authority on incomes and assets of individuals and businesses in compliance with the law, regardless of the exact amount of the benefit the payer gets in return [Gaudemet, Molinier, 2000, p. 407]. Taxation is not for a direct exchange of goods and/or services, but for transferring revenues and incomes from the private sector into the public one to achieve particular economic and social goals of the country [Gomułowicz, Małecki, 2004, p. 126]. Such goals may arise from a high level of employment, stable prices, fast economic growth, a favourable balance of payments, promoting market economy, meeting collective needs, fair income distribution, promoting small businesses, encouraging entering priority sectors, fostering sustainable development of people, and supporting employment and capital. It shows clearly what effect taxes have on the incomes, consumption, production and behaviour of people [Szczodrowski, 2007, pp. 38–39].

The main tax objective is to increase revenues to cover government spending as well as wealth redistribution and economic governance. They are meant to generate revenues for the government, govern economy and business activity and control both revenues and employment [Ustawa z dnia 27 sierpnia 2009 r.]. Moreover, taxes enact the functions of allocation, distribution and stabilization. Taxation is connected with determining the kind of production and goods to be produced, the people who manufacture them, the relationship between the private and public sectors and the point of social balance between those sectors. Tax distribution refers to the effective way in which a demand for economic goods is distributed among particular members of society [Owsiak, 2005, p. 189].

The aim of this study is to determine the relationship between tax revenues and the gross domestic product, investments, government expenditure, exports and imports using a Vector Error Correction (VEC) model. The macroeconomic indicators come from the Macroeconomic Data Bank (BDM) run by the Central Statistical Office. For this article, the VEC model was chosen as a method for analysing the relationships of multidimensional quarterly time series in the years 2002–2018. Therefore, the study not only focuses on economic growth, but also, based on the Granger causality, it shows connections between particular variables that describe tax revenues in Poland. The study used a number of tests, including the Augmented Dickey–Fuller (ADF) test, the Granger causality test and the Johansen cointegration test. Also, the Vector Autoregression (VAR) model was used. All the necessary calculations were made by means of R software.

1. Steps in model construction

The assumption in the analysis of the VEC model is that all independent variables have to be stationary. A few tests can be performed to verify whether the data are stationary, one being the ADF test [Kusideł, 2000, p. 22]. This test is based on the estimated regression equation as follows:

$$\Delta Y_t = \alpha_0 + \gamma Y_{t-1} + \beta_i \sum_{i=1}^{p} \Delta Y_{t-i+1} + \varepsilon_t$$

Johansen's method is based on a trace test and maximum eigenvalue test. The null hypothesis in the trace test means there is no cointegration vector, while the alternative hypothesis means that there are two cointegration vectors. The maximum eigenvalue test examines the null hypothesis that assumes there is no cointegration vector, and the alternative hypothesis means that there is one cointegration vector [Osińska, 2006, p. 219]. Cointegration is one of the ways to avoid the problem of false regression. To find cointegration, Johansen's test was used:

$$LR(r_0) = -T \sum_{j=r_0}^{m} ln(1 - \lambda_i)$$

Where:
m – the number of variables,
T – the number of observations,
λ_i – eigenvalues,
r_0 – rank cointegration tested.

Then a lag was selected for a model to be used to estimate VEC. It is important to determine optimal lag in the analysis using a VEC method because when a set of lags is too long or too short, it will result in a wrong specification of the model. To determine an optimal lag, the following criteria can be used Final Prediction Error (FPE), Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC), and Hannan–Quinn Information Criterion (HQ) [Osińska, 2006, p. 54]. Mathematical values AIC, SIC and HQ can be found by means of the following formulas:

$$AIC = -2\left(\frac{1}{T}\right) + 2(k + T)$$
$$SIC = -2\left(\frac{1}{T}\right) + k\frac{\log(T)}{T}$$
$$HQ = -2\left(\frac{1}{T}\right) + 2k\,log\left(\frac{\log(T)}{T}\right)$$

In the analysis of time series, a question is often posed of whether one economic variable may help forecast another variable. Granger causality may indicate if a variable is two-direction or one-direction. Such a test is conducted to determine the impact of one variable on another, and it should be understood in the context of correlation between the investigated economic processes [Osińska, 2006, pp. 208–209]. The definition of Granger causality is: "If variable X is the cause of variable Y, then an error of Y forecast based on historical values of both variables is

smaller than an error of the forecast based on the historical value of variable Y only"
[Granger, 1980, pp. 329–352].

Generally, the econometric model of time series is a structural model because
it is based on existing economic theory. VAR models were introduced by Christopher A. Sims in 1980 as an alternative for macroeconomic analysis. They draw
on the assumptions of both traditional econometrics and the models of time series
for stationary variables [Kusideł, 2000, pp. 9–11]. Granger presented the research
on stochastic trends in economic variables in 1981. Then Engle and Granger proposed a concept of cointegration and the Error Model (ECM), which allows reading
a cointegration vector as a long-term relationship between variables [Engle, Granger,
1987, p. 251]. Following these events, in 1990, the Johansen–Juselius method was
presented to estimate a long-term cointegration vector with VEC [Johansen, Juselius,
1990, pp. 169–210].

The VEC model is a model of econometric analysis that can be utilised to determine a short-term behaviour of a variable in the long term, due to a long-run shock.
The model offers an easy working procedure, separating long-term components and
short-term components from the process of generating data [Kusideł, 2000, p. 12].
Thus, the VEC model is different from the VAR model in that the VEC model can
be applied to model the data of cointegrated time series [Stempińska et al., 2007,
pp. 391–392]. A general VEC model can be presented as follows:

$$\Delta X_t = \Psi_0 D_t + \sum_{i=1}^{k-1} \Pi_i \Delta X_{t-1} + \Pi X_{t-1} + \xi_t$$

Where:

Π – coefficient matrix including effects of short-term adjustments and long-term cointegration relationships,

Ψ_0 – coefficient matrices with deterministic components of a vector,

Π_i – autoregressive coefficient matrices, and

ξ_t – white noise process [Kusideł, 2000, p. 51].

2. Results of estimation and discussion

Data from the first quarter of 2002 to the first quarter of 2018 were used. They
referred to tax revenue (TAX), gross domestic product (GDP), government expenditure (GOV), investments (INV), exports (EX), and imports (IMP). Following the
commonly adopted practice, the data were expressed in logarithms to smooth them.
Then the analysis was started to investigate whether the variables belong to stationary
or non-stationary series. Table 1 presents descriptive statistics of the collected data.

Table 1. Values of selected variables used in the study

	TAX	GDP	GOV	CON	INV	EX	IMP
Minimum	2.855,8	18.943,2	4.771,2	16.982,8	2.405,6	4.853,1	5.626,7
Quartile Q1	6.248,8	26.394,6	8.753,1	21.117,6	4.796,1	9.419,4	9.673,1
Median	12.217,6	35.707,7	15.960,7	28.608,1	6.804,7	13.187,8	13.969,9
Mean	13.072,6	34.829,7	17.050,0	27.629,7	7.212,5	15.069,1	15.081,6
Quartile Q3	17.858,3	42.484,2	23.163,5	33.410,9	8.454,4	19.917,5	19.151,6
Maximum	31.530,9	55.866,6	37.586,8	39.489,7	15.006,4	27.883,8	26.057,5

Source: Author's own study with the use of R software.

The basic assumption in the construction of the VEC model, which is to be met, is that all the independent variables have to be stationary. To this purpose, the ADF test was used for each variable.

Table 2. Results of ADF tests on stationary for examined series

Variables	Levels		First differences
	Stat. ADF	p-value	Stat. ADF
TAX	-15.2916	2.2e-16	-118.4417
GDP	3.2758	8.439e-08	-32.436
GOV	-18.8052	2.2e-16	-168.8196
CON	-0.9202	0.03061	-7.5387
INV	-6.6758	1.165e-12	-39.2799
EX	-2.7274	0.0003024	-9.2244
IMP	-2.3427	0.01766	-7.0358

ADF critical values at the level of 0.05 is -3.45; for differences: -2.89.

Source: Author's own study with the use of R software.

The results in Table 2 suggest that logarithmic GDP and CON are integrated variables of order 1, while other variables are stationary because statistical values for variables and first differences are below critical values, where $\alpha = 0.05$ is -3.45, and for first differences -2.89.

The presence or lack of cointegration can be proved based on the values of the statistical test (LR_{tr}). Its value is greater than the critical value ($\mathfrak{T}_\sigma$), so cointegration of many variables occurs.

Table 3. Results of the Johansen cointegration test

Null hypothesis (H0)	Trace test (LR_{tr})	0.05 Trace historical value ($\mathfrak{T}_\sigma$)
r<=6	5.51	9.24
r<=5	15.58	19.96
r<=4	33.39	34.91
r<=3	55.1	53.12
r<=2	87.65	76.07
r<=1	130.24	102.14
r=0	183.66	131.7

Source: Author's own study with the use of R software.

The results of Johansen cointegration test proved that there are three integrating vectors. After verifying the existence of long-run relations, a VEC model was constructed. In the next step, the Granger causality test was applied to the examined variables.

Table 4. Results of the Granger causality test

Causality	F	p
TAX → GDP	207.37	2.20e-16
GDP → TAX	136.56	2.20e-16
TAX → GOV	4.514	0.006687
GOV → TAX	2.5156	0.06771
TAX → CON	101.55	2.20e-16
CON → TAX	13.142	1.39e-06
TAX → INV	118.55	2.20e-16
INV → TAX	92.827	2.20e-16
TAX → EX	70.182	2.20e-16
EX → TAX	7.9505	0.0001709
IMP → TAX	113.2	2.20e-16
IMP → TAX	10.56	1.38e-05

Source: Author's own study with the use of R software.

Among the presented factors (components), Granger causality does not occur between government expenditure and tax revenue. It should be pointed out that, according to Granger causality, all the variables are a cause for tax revenue. The optimal number of lags was subsequently determined (k), which are to be applied to estimate VEC by means of information criteria AIC (Akaike), HG (Hannan–Quinn), SIC (Schwarz Information Criterion) and FPE. It is essential to determine an optimal number of lags in the analysis with the use of the VEC method because if a set of lags is too long or too short, it will result in the wrong specification of the model. Table 5 shows an optimal length of lags.

Table 5. Optimal results of determining lag lengths

Lag	AIC	HQ	SIC	FPE
1	-4.54e+01	-4.47e+01	-4.35e+01	1.85e-20
2	-4.78e+01	-4.64e+01	-4.41e+01	1.82e-21
3	**-4.91e+01**	**-4.70e+01**	**-4.37e+01**	**6.10e-22**

Source: Author's own study with the use of R software.

Based on the specification of the model and estimating an optimal number of lags, the best model of the data on tax revenue, VEC (3), was achieved. Only this model proved to be stable. The models were verified in terms of correlation of rests by means of so-called Portmanteau test and White test on heteroscedasticity. Table 6 presents the results.

Table 6. Verification of the statistical model

Assumptions	Test	Chi-squared	p-value
Normality	Jarque–Bera	9.8551	0.77270
Autocorrelation	Portmanteau	687.5400	0.08095
Heteroscedasticity	White	1242.4540	0.41160

Source: Author's own study with the use of R software.

The results of the Jarque–Bera test indicate that there is a relationship of $p > \alpha$ at the level of $\alpha = 0.05$. Therefore, there are no grounds to reject the null hypothesis. The distribution of a random component is a normal distribution. The remaining tests confirm that the model was estimated correctly because there is no autocorrelation and heteroscedasticity.

Based on the model specification and making use of an optimal number of lags, the best model of the data on tax revenue, VEC (3), was achieved. The tax revenue model VEC (3) is as follows:

$$\Delta tax = -0,20 EC_1 - 0,67 EC_2 + 0,21 EC_3 - 0,3\Delta tax_{t-1} + 0,02\Delta gdp_{t-1} + 0,82\Delta gov_{t-1}$$
$$- 0,55\Delta con_{t-1} + 0,21\Delta inv_{t-1} + 0,36\Delta ex_{t-1} - 0,12\Delta imp_{t-1} - 0,28\Delta tax_{t-2}$$
$$+ 1,21\Delta gdp_{t-2} + 0,72\Delta gov_{t-2} - 0,53\Delta con_{t-2} - 0,05\Delta inv_{t-2} - 0,25\Delta ex_{t-2}$$
$$+ 0,38\Delta imp_{t-2} - 0,28\Delta tax_{t-3} + 0,8\Delta gdp_{t-3} + 0,7\Delta gov_{t-3} + 0,01\Delta con_{t-3}$$
$$- 0,06\Delta inv_{t-3} + 0,13\Delta ex_{t-3} + 0,25\Delta imp_{t-3}$$

As the achieved results show, the majority of parameters are statistically significant, the rest do not include autocorrelation, and the adjustment to the empirical data is good. The results of the model show the value of the variable EC_1 at the level of -0.2, which means that around 20% deviation from the long-term path of tax revenue is adjusted by a short-term process of adjustments. Subsequently, EC_2 and EC_3 were -0.67 and 0.21, respectively, which means that return to the state from the previous period, first by -67% and then by 21%, of the deviation from a long-term path of tax revenue is adjusted by a short-term process of adjustments.

Conclusions

With the use of the Granger causality, it was possible to determine the direction of the influence of macroeconomic variables on the tax revenues in the years 2002–2018. Among the analysed factors, Granger causality does not occur only between the government expenditure and tax revenue. The increase in the revenues is contributed by the gross domestic product and the government expenditure in the whole period and by investments and exports in the initial period. This is, however, at the cost of consumption in the whole period and investments at the later stage. The impact of macroeconomic variables on the tax revenue varied at each stage. It is illustrated by imports that react negatively at the beginning, but at the later stage, they are brought

back to balance. On the other hand, one can observe weak investments at the same stage. These changes may result from frequent regulatory changes in taxation or from other one-off economic events. Further analyses are going to be extended with the impact of amendments in tax regulations or other social and economic factors.

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Wpływ zmiennych makroekonomicznych na dochody podatkowe – badanie empiryczne przy użyciu modelu VEC

Dochody podatkowe są ważną determinantą wzrostu i rozwoju gospodarczego każdego kraju. Istotne jest poznanie oddziaływania produktu krajowego brutto, wydatków rządowych, inwestycji, eksportu i importu na wpływy podatkowe w Polsce. Biorąc pod uwagę istotne znacznie dochodów podatków i postać danych jako kwartalne szeregi czasowe z lat 2002–2018, wybrano model VEC jako metodę badania. Miało ono na celu opracowanie modelu VEC z optymalnym opóźnieniem do analizy zależności między dochodami podatkowymi a produktem krajowym brutto, inwestycjami, wydatkami rządowymi, eksportem i importem.

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Tax revenues are an important determinant of the growth and economic development of each country. It is important to learn about the impact of gross domestic product, government expenditure, investments, exports, and imports on tax revenues in Poland. Considering statistically significant tax revenues and the form of data as the quarterly time series from the years 2002–2018, the VEC model was selected as a test method. This study aims to develop a VEC model with an optimal lag to analyse the relationship between tax revenues and gross domestic product, investments, government expenditure, exports, and imports.